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A THERMAL-SHOCK PROBLEM IN MAGNETO-THERMOELASTICITY WITH THERMAL RELAXATION

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Abstract—The one-dimensional problem of distribution of thermal stresses and temperature is considered in a generalized thermoelastic electrically conducting half-space permeated by a primary uniform magnetic field when the bounding plane is suddenly heated to a constant temperature.

The Laplace transform technique is used to solve the problem. Inverse transforms are obtained in an approximate manner using asymptotic expansions valid for small values of time.

Numerical computations for two particular cases are carried out. Copyright © 1996 Elsevier Science Ltd

NOMENCLATURE

T	absolute temperature
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
$\mathbf{H}$	magnetic intensity vector $=(0, H_o, 0)$
$\mathbf{E}$	electric intensity vector
ρ	density
k	thermal conductivity
σ_o	electric conductivity
μ_o	magnetic permeability
λ, μ	Lame's constants
α_t	coefficient of linear thermal expansion
C_E	specific heat for processes with invariant strain tensor
η_o	$= \rho C_E/k$
$c_1 = [(\lambda+2\mu)/\rho]^{1/2}$	speed of propagation of longitudinal isothermal waves
γ	$= (3\lambda+2\mu)\alpha_t$
T_o	reference temperature chosen such that $\lvert(T-T_o)/T_o\rvert \ll 1$
β	$= [(\lambda+2\mu)/\mu]^{1/2}$
b	$= \gamma T_o/\mu$
g	$= \gamma/\rho C_E$

INTRODUCTION

Biot (1956) formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. Lord and Shulmann (1967) introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier's law. This law contains the heat flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and the constitutive relations remain the same as those for the coupled and the uncoupled theories. This theory was extended by Dhaliwal and Sherief (1980) to general anisotropic media in the presence of heat sources.

An increasing attention is being devoted to the interaction between magnetic fields and strain in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics and related topics. Usually, in these investigations the heat equation under consideration is taken as the uncoupled or the coupled equation not the generalized one. This attitude is justified in many situations since the solutions obtained using any of these equations differ little quantitively. However, when short time effects are considered, the full generalized system of equations has to be used or a great deal of accuracy is lost.

A comprehensive review of the earlier contributions to the subject can be found in Paria (1967). Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Nasser (1972) who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They have obtained the governing equations in the general case and the solution for some particular cases. Choudhuri (1984) extended these results to rotating media.

FORMULATION OF THE PROBLEM

We shall consider a homogeneous, isotropic, thermoelastic solid of finite conductivity σ_o occupying the region $x \geqslant 0$, where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. A constant magnetic field with components $(0, H_o, 0)$ is permeating the medium in the absence of an external electric field.

It is assumed that the state of the medium depends only on x and t and that the displacement vector has components $(u(x, t), 0, 0)$. Since no external electric field is applied, and the effect of polarization of the ionized medium can be neglected, it follows that the total electric field $\mathbf{E}$ vanishes identically inside the medium. It can be easily seen from the governing equations in Nayfeh and Nasser (1982) that when the electric field vanishes then the coefficient connecting the temperature gradient and the electric current as well as the coefficient connecting the current density with the heat flow density can be ignored.

The equations of motion in the absence of body forces have the form (see e.g. Nayfeh and Nasser (1982))

$$\rho\frac{\partial^2 u_i}{\partial t^2} = (\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma T_{,i} + (\mathbf{j}\times\mathbf{B})_i, \tag{1}$$

where t denotes the time variable, $\mathbf{B}$ is the magnetic induction vector given by

$$\mathbf{B} = \mu_o \mathbf{H},$$

and $\mathbf{j}$ is the conduction current density, given by Ohm's law

$$\mathbf{j} = \sigma_o\left[\mathbf{E} + \frac{\partial \mathbf{u}}{\partial t}\times\mathbf{B}\right].$$

The constitutive equations are given by

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma(T - T_o)\delta_{ij}, \tag{2}$$

where e_{ij} are given by

$$e_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}).$$

The energy equation in the absence of heat sources can be written as

$$kT_{,ii} = \left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)[\rho C_E T+\gamma T_o e_{kk}], \tag{3}$$

where τ_o is the relaxation time. In the above equations a comma denotes material derivatives and the summation convention is used.

The components of the electromagnetic induction vector are given by

$$B_x = B_z = 0, \quad B_y = \mu_o H_o = B_o(\text{constant}),$$

while the components of $\mathbf{F} = \mathbf{j}\times\mathbf{B}$ appearing in eqn (1) are given by

$$F_x = -\sigma_o B_o^2\frac{\partial u}{\partial t}, \quad F_y = F_z = 0.$$

Using the above values, eqns (1)–(3) reduce to

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\gamma\frac{\partial T}{\partial x}-\sigma_o B_o^2\frac{\partial u}{\partial t}, \tag{4}$$

$$\sigma = \sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x}-\gamma(T-T_o), \tag{5}$$

$$k\frac{\partial^2 T}{\partial x^2} = \rho C_E\left(\frac{\partial T}{\partial t}+\tau_o\frac{\partial^2 T}{\partial t^2}\right)+\gamma T_o\left(\frac{\partial^2 u}{\partial x\,\partial t}+\tau_o\frac{\partial^3 u}{\partial x\,\partial t^2}\right). \tag{6}$$

The governing equations can be put into a more convenient form by using the following non-dimensional variables

$$x' = c_1\eta_o x, \quad u' = c_1\eta_o u, \quad t' = c_1^2\eta_o t, \quad \tau_o' = c_1^2\eta_o\tau_o,$$

$$\theta = (T-T_o)/T_o, \quad \sigma' = \sigma/\mu \quad \text{and} \quad M = \sigma_o B_o^2/\rho c_1^2\eta_o.$$

In terms of these variables, eqns (4)–(6) become (dropping the primes for convenience)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}-a\frac{\partial\theta}{\partial x}-M\frac{\partial u}{\partial t}, \tag{7}$$

$$\sigma = \beta^2\frac{\partial u}{\partial x}-b\theta, \tag{8}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial\theta}{\partial t}+\tau_o\frac{\partial^2\theta}{\partial t^2}+g\left(\frac{\partial^2 u}{\partial x\,\partial t}+\tau_o\frac{\partial^3 u}{\partial x\,\partial t^2}\right), \tag{9}$$

where $a = b/\beta^2$.

The boundary conditions are assumed to be

$$\sigma(x,t)|_{x=0} = 0, \quad \sigma(x,t)|_{x=\infty} = 0, \tag{10}$$

$$\theta(x,t)|_{x=0} = \theta_o\eta(t), \quad \theta(x,t)|_{x=\infty} = 0, \tag{11}$$

where $\eta(t)$ denotes the Heaviside unit step function. These boundary conditions mean that the bounding plane of the half-space is taken as traction free and is subjected to a constant thermal shock at time $t = 0$.

The initial conditions are taken as

$$u(x,t)\big|_{t=0} = \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = 0, \tag{12}$$

$$\theta(x,t)\big|_{t=0} = \left.\frac{\partial \theta(x,t)}{\partial t}\right|_{t=0} = 0. \tag{13}$$

SOLUTION IN THE LAPLACE TRANSFORM DOMAIN

We now introduce the Laplace transform defined by the formula

$$\bar{f}(p) = \int_0^\infty e^{-pt} f(t)\,\mathrm{d}t. \tag{14}$$

Applying this transform to eqns (7)–(9), and using the initial conditions (12) and (13), we obtain

$$\left(\frac{\partial^2}{\partial x^2} - Mp - p^2\right)\bar{u} = a\frac{\partial \bar{\theta}}{\partial x}, \tag{15}$$

$$\bar{\sigma} = \beta^2 \frac{\partial \bar{u}}{\partial x} - b\bar{\theta}, \tag{16}$$

$$\left(\frac{\partial^2}{\partial x^2} - p - \tau_o p^2\right)\bar{\theta} = g(p + \tau_o p^2)\frac{\partial \bar{u}}{\partial x}. \tag{17}$$

Introducing the thermoelastic potential function ϕ defined by the relation

$$u = \frac{\partial \phi}{\partial x}, \tag{18}$$

eqns (15)–(17) reduce to

$$\left(\frac{\partial^2}{\partial x^2} - Mp - p^2\right)\bar{\phi} = a\bar{\theta}, \tag{19}$$

$$\bar{\sigma} = \beta^2 \frac{\partial^2 \bar{\phi}}{\partial x^2} - b\bar{\theta}, \tag{20}$$

$$\left(\frac{\partial^2}{\partial x^2} - p - \tau_o p^2\right)\bar{\theta} = g(p + \tau_o p^2)\frac{\partial^2 \bar{\phi}}{\partial x^2}. \tag{21}$$

The boundary conditions (10) and (11) expressed in terms of $\bar{\phi}$ take the form

$$\bar{\phi}(x,p)|_{x=0} = 0, \quad \bar{\phi}(x,p)|_{x=\infty} = 0, \tag{22}$$

$$\left.\frac{\partial^2 \bar{\phi}(x,p)}{\partial x^2}\right|_{x=0} = \frac{a\theta_o}{p}, \quad \left.\frac{\partial^2 \bar{\phi}(x,p)}{\partial x^2}\right|_{x=\infty} = 0. \tag{23}$$

Eliminating $\bar{\theta}$ between eqns (19) and (21), we obtain the following equation satisfied by $\bar{\phi}$

$$\left[\frac{\partial^4}{\partial x^4} - p[(1+\varepsilon)(1+\tau_o p)+M+p]\frac{\partial^2}{\partial x^2} + p^2(M+p)(1+\tau_o p)\right]\bar{\phi} = 0, \tag{24}$$

where ε is a positive constant defined by $\varepsilon = ag$.

The general solution of eqn (24) for $x \geqslant 0$ is

$$\bar{\phi} = A_1 e^{-k_1 x} + A_2 e^{-k_2 x}, \tag{25}$$

where A_1 and A_2 are parameters depending on p to be determined from the boundary conditions and k_1 and k_2 are the roots with positive real parts of the characteristic equation

$$k^4 - p[(1+\varepsilon)(1+\tau_o p)+p+M]k^2 + p^2(M+p)(1+\tau_o p) = 0, \tag{26}$$

k_1 and k_2 are given by

$$k_{1,2} = \left\{\frac{p}{2}[(M+p)+(1+\varepsilon)(1+\tau_o p) \pm ((M+p)^2 + 2(\varepsilon-1)(M+p)(1+\tau_o p) + (1+\varepsilon)^2(1+\tau_o p)^2)^{1/2}]\right\}^{1/2}. \tag{27}$$

From the boundary conditions (22) and (23), it follows that

$$A_1 = -A_2 = \frac{a\theta_o}{p(k_1^2 - k_2^2)}, \tag{28}$$

hence

$$\bar{\phi} = \frac{a\theta_o}{p(k_1^2 - k_2^2)}\{e^{-k_1 x} - e^{-k_2 x}\}, \tag{29}$$

$$\bar{\theta} = \frac{\theta_o}{p(k_1^2 - k_2^2)}\{[k_1^2 - p(M+p)]e^{-k_1 x} - [k_2^2 - p(M+p)]e^{-k_2 x}\}, \tag{30}$$

$$\bar{\sigma} = \frac{b\theta_o(M+p)}{k_1^2 - k_2^2}\{e^{-k_1 x} - e^{-k_2 x}\}.$$

INVERSION OF THE LAPLACE TRANSFORMS

Let us now determine inverse transforms for the case of small values of time (large values of p). Denoting $u = p^{-1}$, we have

$$k_i = u^{-1}[f_i(u)]^{1/2}, \quad i = 1, 2, \tag{32}$$

where

$$f_{1,2}(u) = \tfrac{1}{2}\{1+Mu+(1+\varepsilon)(u+\tau_o) \pm [(1+Mu)^2+2(\varepsilon-1)(1+Mu)(u+\tau_o)+(1+\varepsilon)^2(u+\tau_o)^2]^{1/2}\}.$$

Expanding $f_1(u)$ and $f_2(u)$ in a Maclaurin series of which the first four terms are retained, we get

$$f_i(u) = \sum_{j=0}^{3} a_{ij}u^j, \quad i = 1,2, \tag{33}$$

where

$$a_{10} = \tfrac{1}{2}[1+(\varepsilon+1)\tau_o+A],$$
$$a_{11} = \tfrac{1}{2}[M+\varepsilon+1+B/A],$$
$$a_{12} = \frac{A^2C-B^2}{4A^3},$$
$$a_{13} = \frac{B(B^2-A^2C)}{4A^5},$$
$$a_{20} = \tfrac{1}{2}[1+(\varepsilon+1)\tau_o-A],$$
$$a_{21} = \tfrac{1}{2}[M+\varepsilon+1-B/A],$$
$$a_{22} = -a_{12}, \quad a_{23} = -a_{13},$$

and

$$A = [1+2(\varepsilon-1)\tau_o+(\varepsilon+1)^2\tau_o^2]^{1/2},$$
$$B = M+(\varepsilon-1)(1+M\tau_o)+(\varepsilon+1)^2\tau_o,$$
$$C = (\varepsilon+M)^2-2(M-\varepsilon)+1.$$

Next, we expand the expressions $[f_i(u)]^{1/2}$ in a Maclaurin series and, retaining the first three terms, we obtain the expressions for k_i in the form

$$k_i = u^{-1}\sum_{j=0}^{2} b_{ij}u^j, \quad i = 1,2, \tag{34}$$

where

$$b_{i0} = \sqrt{a_{i0}},$$
$$b_{i1} = \frac{a_{i1}}{2\sqrt{a_{i0}}},$$
$$b_{i2} = \frac{1}{8a_{i0}^{3/2}}(4a_{i2}a_{i0}-a_{i1}^2).$$

Using similar expansion methods, we obtain

$$\frac{1}{k_1^2-k_2^2} = u^2\sum_{j=0}^{3} b_j u^j, \tag{35}$$

where

$$b_0 = \frac{1}{A},$$

$$b_1 = -\frac{B}{A^3},$$

$$b_2 = \frac{3B^2 - CA^2}{2A^5},$$

$$b_3 = \frac{B(3CA^2 - 5B^2)}{2A^7},$$

STRESS DISTRIBUTION

Let us substitute the expressions (34) and (35) into eqn (31) to obtain

$$\bar{\sigma} = b\theta_o \sum_{i=1}^{2} \sum_{j=0}^{3} (-1)^{i+1} \frac{m_j}{p^{j+1}} e^{-x(b_{i0}p + b_{i1} + b_{i2}/p)}, \tag{36}$$

where

$$m_o = b_o, \quad m_j = b_j + Mb_{j-1}, \quad j = 1, 2, 3.$$

In order to invert the Laplace transforms in eqn (36), we shall use the convolution theorem of the Laplace transform (see e.g. Oberhettinger and Badii (1973)), namely

$$\mathscr{L}^{-1}[\bar{f}_1(p)\bar{f}_2(p)] = \int_0^t f_1(t-z) f_2(z)\,\mathrm{d}z,$$

and the following formulae from Oberhettinger and Badii (1973)

$$\mathscr{L}^{-1}(p^{-v-1}) = \frac{t^v}{\Gamma(v+1)}, \quad \operatorname{Re} v > -1,$$

$$\mathscr{L}^{-1}(p^{-v-1}e^{-h/p}) = \left(\frac{t}{h}\right)^{v/2} J_v(2\sqrt{ht}), \quad \operatorname{Re} v > -1, \quad h > 0,$$

$$\mathscr{L}^{-1}(e^{-hp}) = \delta(t-h),$$

$$\mathscr{L}^{-1}(p^{-v-1}e^{h/p}) = \left(\frac{t}{h}\right)^{v/2} I_v(2\sqrt{ht}), \quad \operatorname{Re} v > -1, \quad h > 0,$$

where J_v and I_v are the Bessel and the modified Bessel functions of order v of the first kind, respectively.

The sign of the quantities b_{12} and b_{22} plays an important role in the inversion process. In Sherief and Dhaliwal (1981), it was found that for the same problem in the absence of the applied magnetic field, we always have $b_{12} > 0$ and $b_{22} < 0$. In the presence of the magnetic field, however, it was found that there is a certain value $M = M^*$ depending on the properties of the medium such that

$$b_{12} > 0 \quad \text{for} \quad M < M^*,$$

$$b_{12} = 0 \quad \text{for} \quad M = M^*,$$

$$\text{and} \quad b_{12} < 0 \quad \text{for} \quad M > M^*.$$

The value of b_{22} is always negative as in Sherief and Dhaliwal (1981).

Using the above formulae, we have

$$\mathscr{L}^{-1}(e^{-b_{i0}xp}) = \delta(t-b_{i0}x), \quad i = 1,2,$$

$$\mathscr{L}^{-1}(p^{-j-1}e^{-b_{12}x/p}) = \begin{cases} \left(\dfrac{t}{b_{12}x}\right)^{j/2} J_j(2\sqrt{b_{12}xt}) & \text{for} \quad M < M^*, \\ \dfrac{(t-b_{10}x)^j}{j!} & \text{for} \quad M = M^*, \\ \left(\dfrac{-t}{b_{12}x}\right)^{j/2} I_j(2\sqrt{-b_{12}xt}) & \text{for} \quad M > M^*, \end{cases}$$

$$\mathscr{L}^{-1}(p^{-j-1}e^{-b_{22}x/p}) = \left(\frac{-t}{b_{22}x}\right)^{j/2} I_j(2\sqrt{-b_{22}xt}).$$

Using these expressions together with the convolution theorem in which we take

$$\bar{f}_1(p) = \exp(-b_{i0}xp) \quad \text{and} \quad \bar{f}_2(p) = p^{-v-1}\exp(-b_{i2}x/p),$$

we obtain for $M < M^*$

$$\sigma = b\theta_0 \left\{ e^{-b_{11}x}\eta(x_1) \sum_{j=0}^{3} m_j y_1^{j/2} J_j(z_1) - e^{-b_{21}x}\eta(x_2) \sum_{j=0}^{3} m_j y_2^{j/2} I_j(z_2) \right\}. \tag{37}$$

For $M > M^*$, we have

$$\sigma = b\theta_0 \left\{ e^{-b_{11}x}\eta(x_1) \sum_{j=0}^{3} m_j y_1^{j/2} I_j(z_1) - e^{-b_{21}x}\eta(x_2) \sum_{j=0}^{3} m_j y_2^{j/2} I_j(z_2) \right\}, \tag{38}$$

while for $M = M^*$, we get

$$\sigma = b\theta_0 \left\{ e^{-b_{11}x}\eta(x_1) \sum_{j=0}^{3} m_j \frac{x_1^j}{j!} - e^{-b_{21}x}\eta(x_2) \sum_{j=0}^{3} m_j y_2^{j/2} I_j(z_2) \right\}, \tag{39}$$

where

$$x_i = t - b_{i0}x,$$

$$y_i = \frac{t-b_{i0}x}{|b_{i2}|x},$$

$$z_i = 2\sqrt{x|b_{i2}|(t-b_{i0}x)}.$$

From eqns (37)–(39), it follows that $\sigma(x, t)$ is a continuous function for $0 \leqslant x \leqslant \infty$ except for the points $x = t/b_{10}$ and $x = t/b_{20}$ where jumps of the magnitudes $-b\theta_0 m_0 \exp(-b_{11}t/b_{10})$ and $b\theta_0 m_0 \exp(-b_{21}t/b_{20})$, respectively, occur.

TEMPERATURE DISTRIBUTION

Substituting from (32)–(35) into eqn (30), we obtain

$$\bar{\theta} = \theta_0 \sum_{i=1}^{2} \sum_{j=0}^{3} (-1)^{i+1} \frac{c_{ij}}{p^{j+1}} e^{-x(b_{i0}p+b_{i1}+b_{i2}/p)}, \tag{40}$$

where

$$c_{i0} = b_0(a_{i0}-1),$$

$$c_{ij} = \sum_{k=0}^{j} a_{ik} b_{j-k} - b_j - M b_{j-1}, \quad j = 1,2,3.$$

Using inversion techniques similar to these in the previous section, we obtain for $M < M^*$

$$\theta = \theta_0 \left\{ e^{-b_{11}x} \eta(x_1) \sum_{j=0}^{3} c_{1j} y_1^{j/2} J_j(z_1) - e^{-b_{21}x} \eta(x_2) \sum_{j=0}^{3} c_{2j} y_2^{j/2} I_j(z_2) \right\}. \tag{41}$$

For $M > M^*$, we get

$$\theta = \theta_0 \left\{ e^{-b_{11}x} \eta(x_1) \sum_{j=0}^{3} c_{1j} y_1^{j/2} I_j(z_1) - e^{-b_{21}x} \eta(x_2) \sum_{j=0}^{3} c_{2j} y_2^{j/2} I_j(z_2) \right\}. \tag{42}$$

Finally, for $M = M^*$, we have

$$\theta = \theta_0 \left\{ e^{-b_{11}x} \eta(x_1) \sum_{j=0}^{3} c_{1j} \frac{x_1^j}{j!} - e^{-b_{21}x} \eta(x_2) \sum_{j=0}^{3} c_{2j} y_2^{j/2} I_j(z_2) \right\}. \tag{43}$$

From eqns (41)–(43), it follows that the function θ has two discontinuities at the points $x = t/b_{10}$ and $x = t/b_{20}$. The jumps at these points have magnitudes $\theta_0 \exp(-b_{11}t/b_{10})c_{10}$ and $-\theta_0 \exp(-b_{21}t/b_{20})c_{20}$, respectively.

NUMERICAL RESULTS

The copper material was chosen for purposes of numerical evaluations. The stress and temperature are evaluated for $t = 0.2$. The stress distribution is shown in Fig. 1 while the temperature distribution is shown in Fig. 2. The material constants are taken as

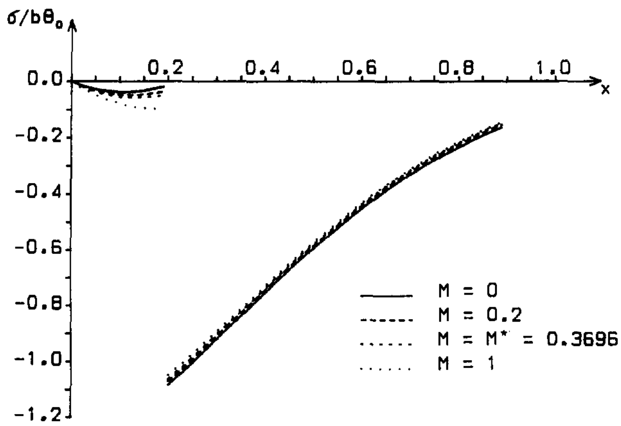

Fig. 1. Stress distribution for $t = 0.2$.

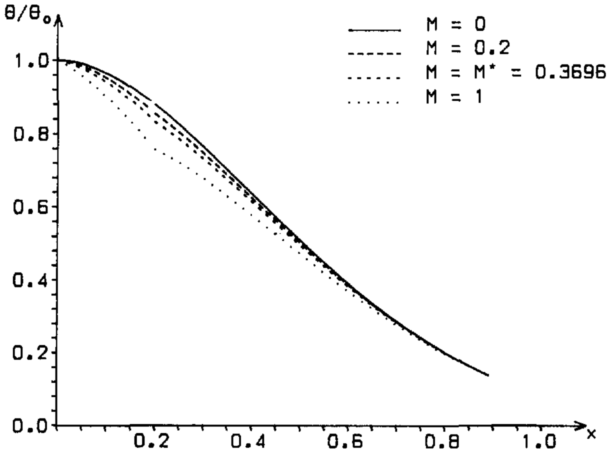

Fig. 2. Temperature distribution for $t = 0.2$.

$$\varepsilon = 0.0168, \quad \beta^2 = 3.5 \quad \text{and}\, \tau_0 = 0.05.$$

The value of M^* (the solution of the equation $b_{12} = 0$) was found, using numerical interpolation techniques, to be 0.3696, approximately. The computations were carried out for four values of M. The first value $M = 0$ gives the solution of the problem in the absence of the applied magnetic field as given in Sherief and Dhaliwal (1981). The second value of M is taken as $M = 0.2$ which is less than M^*. This is the solution for weak magnetic fields as given by eqns (37) and (41) for the stress and temperature distributions, respectively. The third value chosen for M is $M = M^*$ for which the stress and temperature are calculated from eqns (39) and (43), respectively. The fourth and final value taken for M is $M = 1 > M^*$ for which the functions are calculated from eqns (38) and (42).

It is seen from Figs 1 and 2 that the magnetic field acts to decrease the temperature as well as the magnitude of the stress.

The stress and the temperature distributions each has two discontinuties at the locations $x = t/b_{10}$ and t/b_{20}. Since b_{10} and b_{20} are both independent of M the discontinuities occur at the same points for all eight curves. For our case, these values are equal to 0.199912 and 0.894823, respectively. Again, the values of m_0, c_{10} and c_{20} are all independent of M which means that the magnitude of the discontinuities are the same for all curves. For the stress these jumps are equal to -1.04965 and 0.142584, respectively. The jumps in the temperature are equal to 0.000928 and 0.135461, respectively. It is seen thus that the applied magnetic field does not effect either the locations or the magnitudes of the discontinuities.

It is seen that the values of the stress or the temperature are identically zero for $x > t/b_{20}$. Thus, the effect of the thermal shock does not reach infinity instantaneously but remains in a bounded region that expands with the passage of time. The value $x = t/b_{20}$ is the location of the wave front. This is not the case when using the equations of coupled thermoelasticity as in Paria (1967) where thermal effects are felt instantaneously at infinity signifying infinite speeds of propagation for thermal waves.

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